Automatic Data Dependence Analysis by Deductive Verification

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joint work with:
Richard Bubel and Reiner Hähnle

Data Dependence

```
m = a;   //Write m
  ⋮
b = m;   //Read m
```

Data Dependence

```
m = a;   //Write m
  ⋮
b = m;   //Read m
```

Data Dependence

```
m = a;   //Write m
  ⋮
b = m;   //Read m
```

Data Dependence

Read after Write (RaW)

```
m = a;   //Write m
  ⋮
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Data Dependence

Read after Write (RaW)

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m = a;   //Write m
  ⋮
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```

Write after Read (WaR)

```
b = m;   //Read m
  ⋮
m = a;   //Write m
```

Data Dependence

Read after Write (RaW)

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  ⋮
b = m;   //Read m
```

Write after Read (WaR)

```
b = m;   //Read m
  ⋮
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Write after Write (WaW)

```
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  ⋮
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Data Dependence

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Write after Read (WaR)

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Write after Write (WaW)

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Showing noRaW, noWaR, or noWaW $\Rightarrow$ Opportunity for parallelization

Data Dependence

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Analysis

Static $\rightsquigarrow$ Missing Parallelization Opportunities
Dynamic $\rightsquigarrow$ Wrong Behavior

Data Dependence

Read after Write (RaW)

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Write after Read (WaR)

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Showing noRaW, noWaR, or noWaW $\Rightarrow$ Opportunity for parallelization

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Deductive Verification

Program Logic

Access Updates

$$\Longrightarrow \texttt{[a = m; m = b;]post}$$

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$$\Longrightarrow \{\ \texttt{a} := \texttt{m}\ \|\qquad\qquad\ \}[\ \texttt{m = b;}\ ](\texttt{post} \wedge\qquad\qquad)$$

Program Logic

Access Updates

$$\Longrightarrow [\ \texttt{a = m; m = b;}\ ]\,\texttt{post}$$

$$\Longrightarrow \{\ \texttt{a} := \texttt{m} \parallel \texttt{R(m); W(a)}\ \}[\ \texttt{m = b;}\ ](\texttt{post} \wedge \texttt{noRaW(m)})$$

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Trace Semantics

Ghost Variable: $\langle$ (read, m) $\circ$ (write, a) $\rangle$

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Trace Semantics

Ghost Variable: $\langle$ (read, m) $\circ$ (write, a) $\rangle$

noRaW(m)

Ghost Variable: $\langle \cdots \circ$ (write, x) $\circ \cdots \circ$ (read, y) $\circ \cdots \rangle$

$$x \cap y \cap m = \emptyset$$

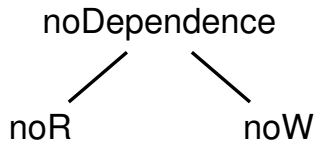
Loop Invariant Generation

Abstraction

Data Dependence	Memory Location
noDependence	$a[l_0 \,..\, h_0]$

Abstraction

Data Dependence



Memory Location

$a[l_0 .. h_0]$

Abstraction

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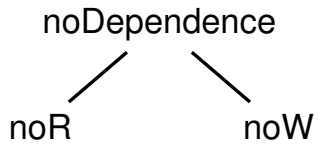

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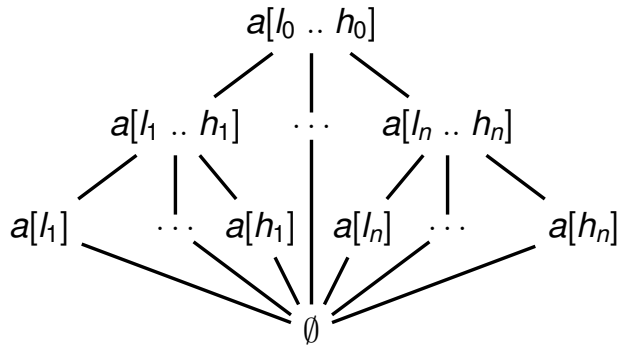

Abstraction

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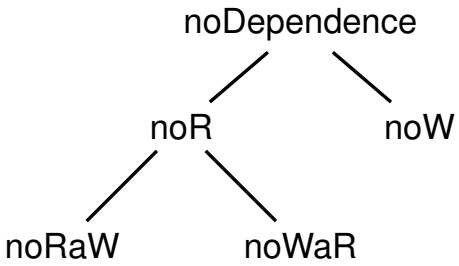

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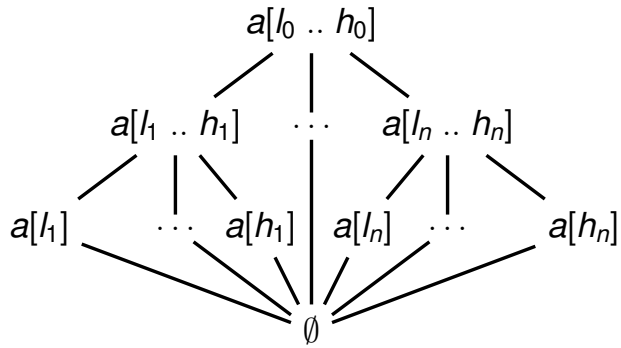

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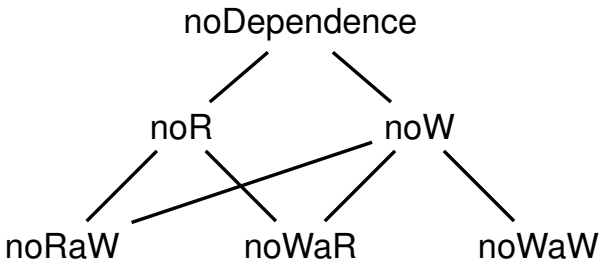

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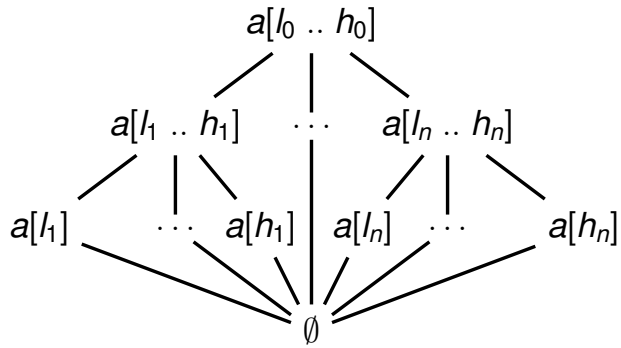

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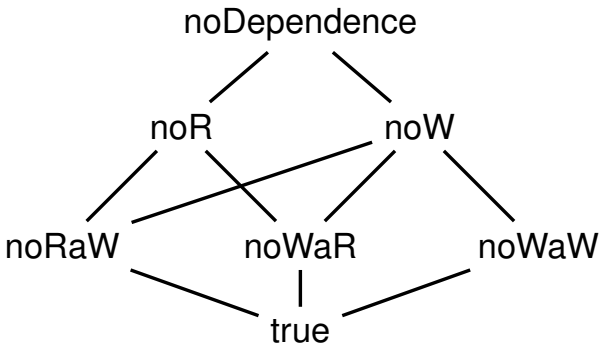

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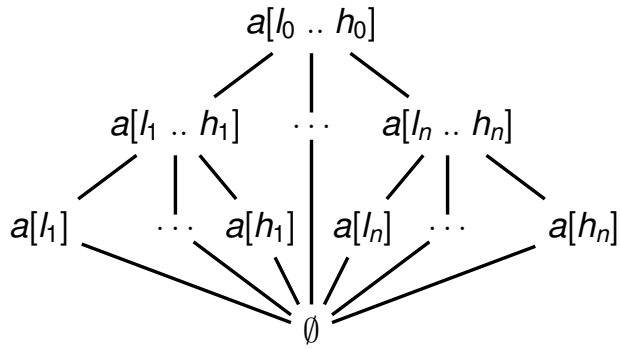

Predicate Abstraction for Loop Invariant Generation

1. Initial set of predicates

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Refining while Symbolically Executing

- Unwind

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- Unwind $\rightsquigarrow$ Ending up with updates on modality

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$$\frac{\{\mathtt{R}^{-1}(\mathtt{m})\}\Gamma,\ \mathtt{Read}(\mathtt{m},0) \Longrightarrow \phi, \{\mathtt{R}^{-1}(\mathtt{m})\}\Delta}{\Gamma \Longrightarrow \{\mathtt{R}(\mathtt{m})\}\phi, \Delta}$$

$\rightsquigarrow$ **After** $\{\mathtt{R}(\mathtt{m})\}$

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$\rightsquigarrow$ **After** $\{\mathtt{R}(\mathtt{m})\}$

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- Evaluate each predicate in the new sequent
- If not proven, weaken

Example

```
int i = 0;
while(i < n - 1) {
   a[i] = a[i+1];
   i++;
}
```

noDependence(a[*])

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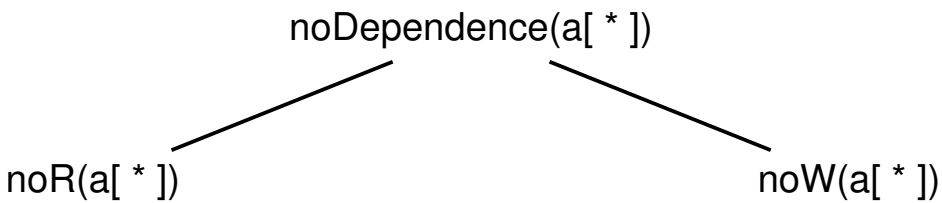

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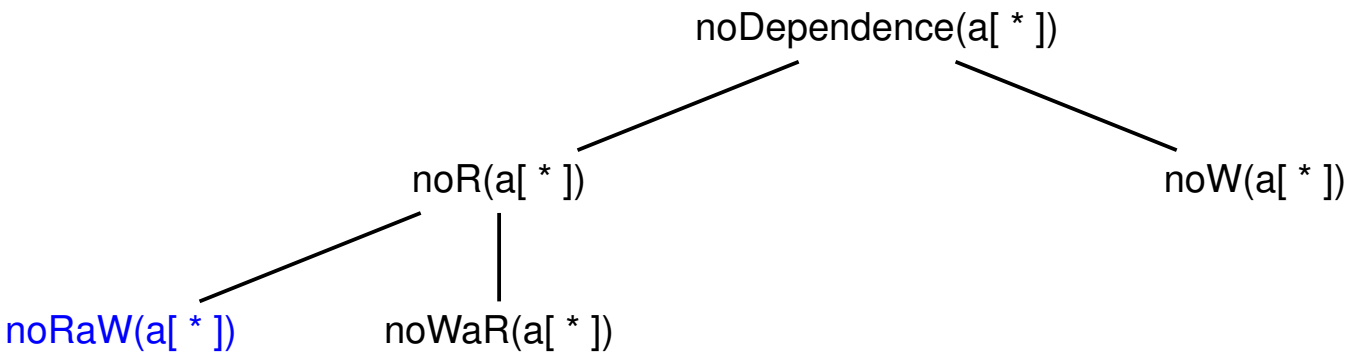

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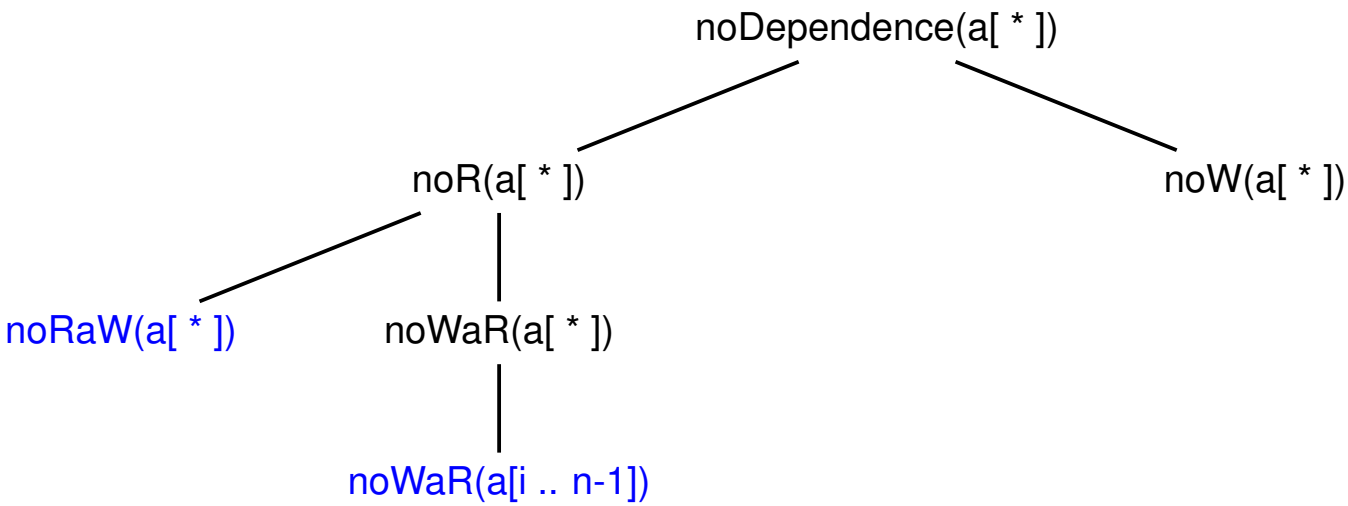

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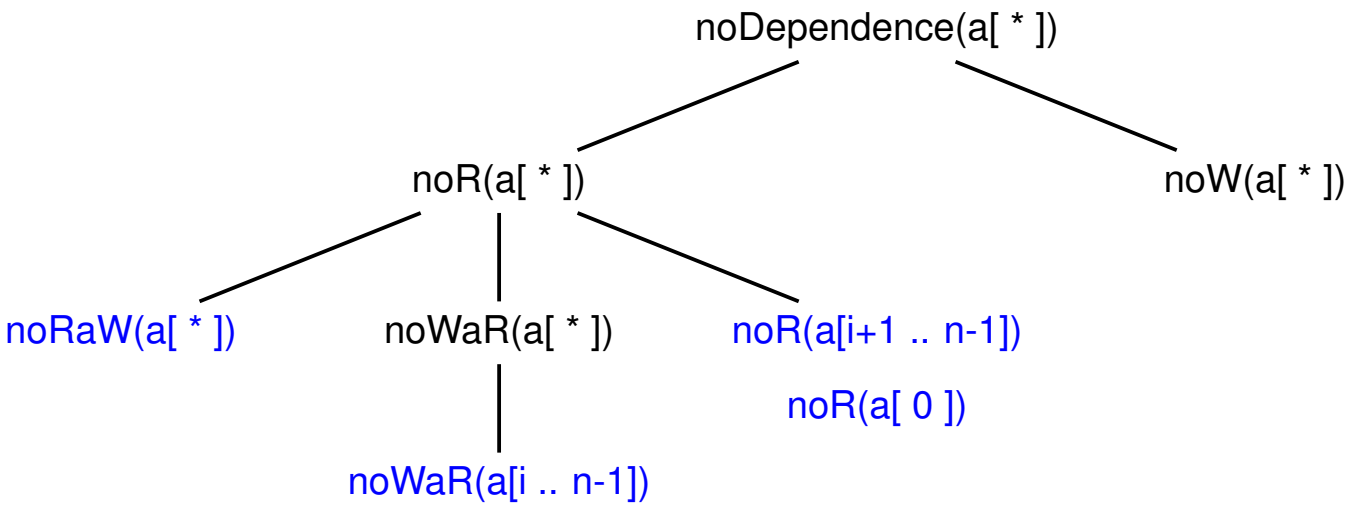

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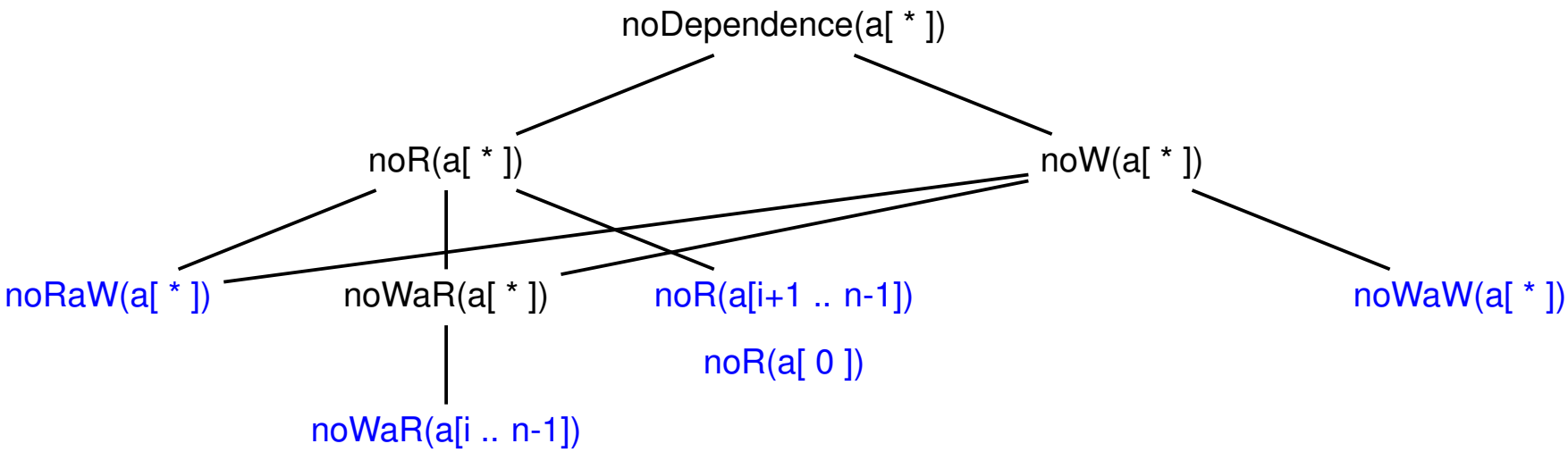

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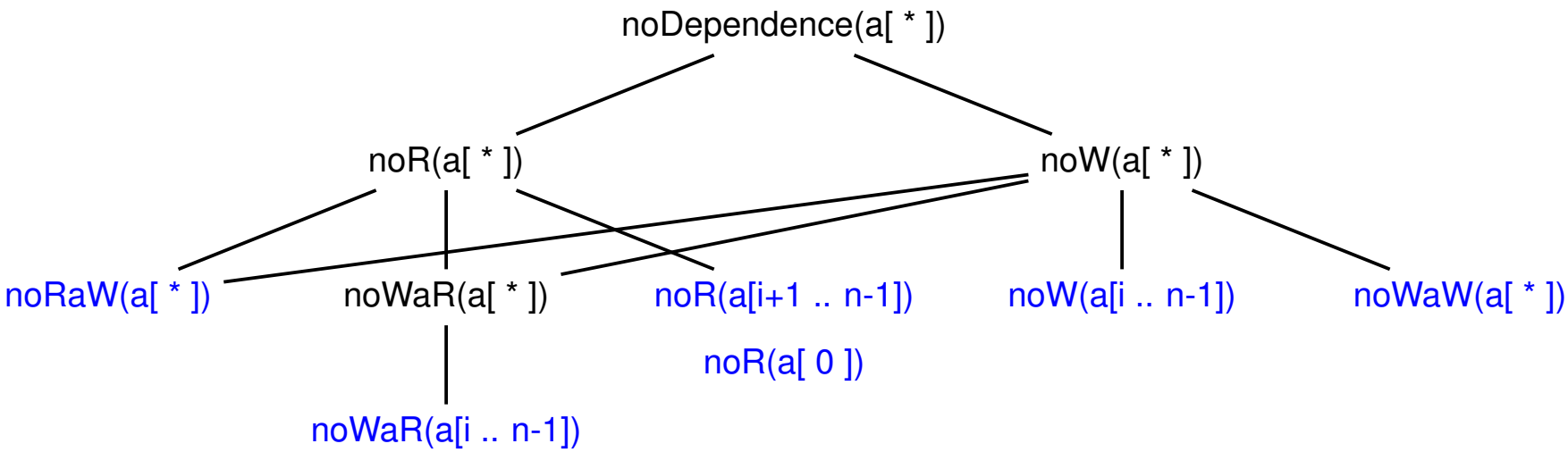

Invariant Generation for Nested Loops

What to do with the Inner Loop?

- Generate invariant for the inner loop
- Replace the inner loop with its invariant
- Apply the Use Case of the loop invariant rule

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$$\frac{\begin{array}{l}\text{Invariant Initially Valid}\\ \text{Body Preserves Invariant}\\ \text{Use Case}\end{array}}{\Gamma \Longrightarrow \{u\}[\texttt{while (}b\texttt{) \{}s\texttt{\} }r\texttt{;}]\phi, \Delta}$$

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Example

```
int i = 0;
int j = 0;
while(i < n - 1) {
   while(j < m - 1) {
      a[i][j] = a[i+1][j+1];
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   }
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}
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      a[i][j] = a[i+1][j+1];
      j++;
   }
   j = 0;
   i++;
}
```

Loop Invariant:

$$
\begin{aligned}
&\text{noW(a[i .. n-1][j .. m-1])} \quad \wedge \\
&\text{noR(a[i+1 .. n-1] [j+1 .. m-1])} \quad \wedge \\
&\text{noR(a[0][0])} \quad \wedge \\
&\text{noWaW(a[*][*])} \quad \wedge \\
&\text{noRaW(a[*][*])} \quad \wedge \\
&0 \leq i \leq n - 1 \quad \wedge \\
&0 \leq j \leq m - 1
\end{aligned}
$$

There is more ...

- No assumption about nested loop

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- Inter-iteration dependences
- Conditionals, aliasing, and function calls
- Precise result for PolyBench-c-4.2.1

Take Away

- Data dependence analysis with high precision
- Weakest Pre Cond. $\rightsquigarrow$ Strongest Post Cond. with Shift Update
- Use cases with an event after another event

Backup

Array Abstraction Heuristic

Backup

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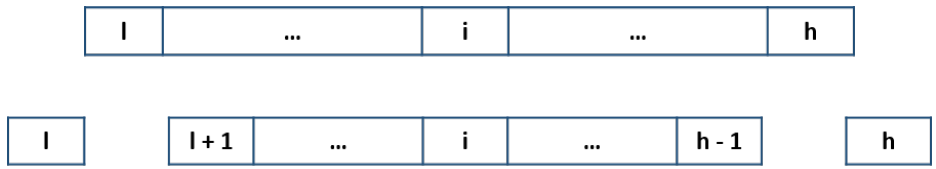

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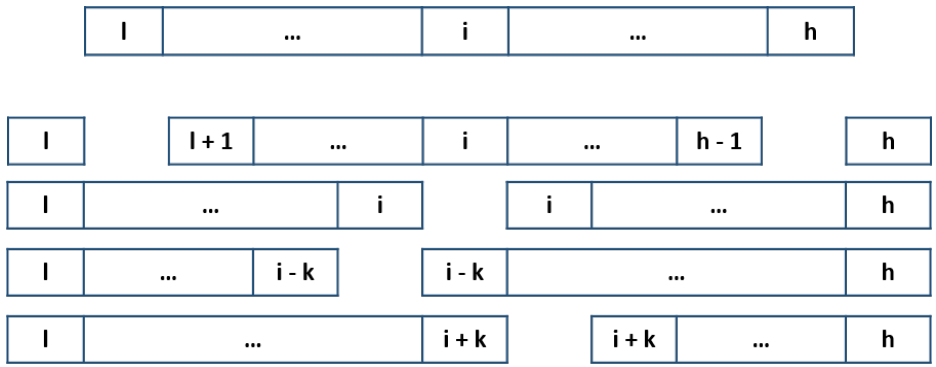

Backup

Shift Update

$$\frac{\{u'\}\Gamma,\ \mathit{link}(u,\ u') \Longrightarrow \phi,\ \{u'\}\Delta}{\Gamma \Longrightarrow \{u\}\ \phi, \Delta}$$

⇝ Evaluated after $\{u\}$

⇝ Evaluated before $\{u\}$

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Example

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Shift Read Access Update

$$\frac{\{\mathtt{R}^{-1}(m)\}\Gamma, \mathtt{Read(m,\ 0)} \Longrightarrow \phi, \{\mathtt{R}^{-1}(m)\}\Delta}{\Gamma \Longrightarrow \{\mathtt{R}(m)\}\phi, \Delta}$$

- ⇝ Evaluated at ghost variable: ..., (read, m)
- ⇝ Evaluated at ghost variable: ...

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0
↓

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Example

$$\frac{\{\mathtt{R}^{-1}(b)\}\Gamma,\ \mathtt{Read(b,\ 0)} \Longrightarrow \phi, \quad \{}{\Gamma \Longrightarrow \{\mathtt{R}(a),\ \mathtt{R}(b)\}\phi, \Delta}$$

Backup

Shift Read Access Update

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$$\frac{\{\mathtt{R}^{-1}(a)\}\{\mathtt{R}^{-1}(b)\}\Gamma,\ \mathtt{Read(b,\ 1), Read(a,\ 0)} \Longrightarrow \phi, \{\mathtt{R}^{-1}(a)\}\{\mathtt{R}^{-1}(b)\}\Delta}{\Gamma \Longrightarrow \{\mathtt{R}(a),\ \mathtt{R}(b)\}\phi, \Delta}$$